VALPARAISO'S MATHEMATICS AND ITS APPLICATIONS DAYS

Noveno Encuentro de la Matemática y sus Aplicaciones

Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso

Valparaíso, 17 y 18 de Enero de 2019

PROGRAMA

1. INTRODUCCIÓN

El **Noveno Encuentro de la Matemática y sus Aplicaciones** ha sido organizado en conferencias secuenciales de **45 y 30 minutos** de duración (40 y 25 minutos de exposición, respectivamente, y 5 minutos para preguntas y comentarios). Todas las charlas se llevarán a cabo en la sala **IMA 2-3** en el Instituto de Matemáticas de la Pontificia Universidad Católica de Valparaíso.

En la siguiente página se detalla la programación correspondiente, incluyendo autor y título de la charla.

La organización agradece al Instituto de Matemáticas y al proycto MathRocks por su gran apoyo para llevar a cabo este evento. Igualmente, extiende su reconocimiento y gratitud a todos los expositores, quienes gracias a su buena voluntad de participar, han hecho posible la realización de este **VMAD 9**.

Comité Organizador

Valparaíso, Enero de 2019

2. JUEVES, 17 DE ENERO

11.10-11.15 BIENVENIDA

11.15-12.00 DAVID PARDO: *A Painless Automatic hp-Adaptive Strategy for Elliptic 1D and 2D Problems: Preliminary Results.*

12.00-12.45 ROLANDO REBOLLEDO: *Diffusion approximation in population dynamics.*

[Moderador: M. Barrientos]

12.45-15.00 ALMUERZO

15.00-15.30 PAULINA SEPÚLVEDA: *Two discontinuous spacetime methods for the linear transport and wave equations.*

15.30-16.00 SERGIO ROJAS: *Automatic variationally stable finite element formulations.*

16.00-16.30 JUDITH MUÑOZ MATUTE: *Goal-oriented adaptivity employing forward-in-time pseudo-dual problems.*

[Moderador: S. Ossandón]

16.30-17.00 COFFEE BREAK

17.00-17.30 NORBERTO SAINZ: *Maxwell Revisited.*

17.30-18.00 IGNACIO BREVIS: *Time reversal methods for source reconstruction on acoustic and elastic waves.*

[Moderador: I. Muga]

3. VIERNES, 18 DE ENERO

11.15-12.00 LUCA GERARDO-GIORDA: *Fractional operators in action: tackling important modeling challenges in cardiac electrophysiology.*

12.00-12.45 DAE-JIN LEE: *Spatial and spatio-temporal modelling for disaggregation of counts in epidemiology.*

[Moderador: M. Barrientos]

12.45-15.00 ALMUERZO

15.00-15.30 ALDONZA JAQUES: *Fractional Calculus approach for anomalous diffusion*

15.30-16.00 KARINA VILCHES: *Simultaneous Blow-up in a Multi-species Keller-Segel system: An energy method.*

16.00-16.30 FELIPE COSTA: *Phenomenological models for species transport and Soret effect during the operation of an OACVD reactor in graphene synthesis.*

[Moderador: S. Rojas]

16.30-17.00 COFFEE BREAK

17.00-17.30 CAMILO REYES: *Just-in-Time Point prediction using a computationally-efficient Lebesgue-sampling-based prognostic method: application to battery end-of-discharge prediction.*

17.30-18.00 JONATHAN ACOSTA: *The effective sample size.*

[Moderador: S. Ossandón]

20:30 CENA DE CAMARADERIA

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Noveno Encuentro en Aplicaciones de la Matemática

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The Effective Sample Size

Jonathan Acosta S. *

Abstract

Griffith (2005)[1] suggested a formula to compute the effective sample size, say n^*, for georeferenced data. In this article, we provide mathematical support that enhances the use of this definition in practice. We prove that $n^* \in [1, n]$ and that n^* is increasing in n. We also prove the asymptotic normality of the maximum likelihood estimate of n^* for an increasing domain sampling framework. Asymptotic normality leads to an approximate hypothesis testing that establishes whether redundant information exists in a sample. A numerical example is presented to illustrate the applicability of the methodology.

References

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Time reversal methods for source reconstruction on acoustic and elastic waves

Ignacio Brevis *

Abstract

Time reversibility is a property widely studied in waves and employed on different applications in physics and engineering, for example in the detection of tumors and kidney stones in medical imaging, detection of defects in metals, and long-distance communication and mine detection in the ocean. On this talk, we present a method, called *source time reversal* [1, 2], that reconstructs the space counterpart of a source $f(\mathbf{x})g(t)$ in non-homogeneous wave equations. Particularly, we show the case of acoustic and elastic waves. This method is based on a known full description of the temporal dependence of the source, the Duhamel's principle, and the time-reverse property of the homogeneous wave equation. We also present some theoretical results and numerical examples. Finally, we show an application to seismicity induced by mining.

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Spatial and spatio-temporal modelling for disaggregation of counts in epidemiology

Dae-Jin Lee *

Abstract

In spatial epidemiology data are usually reported as counts of rates at aggregated level (i.e. municipalities or regions) as well as aggregated over a time period (i.e. monthly or weekly). In this talk, I will present our recent advances in spatial and spatio-temporal modelling via the so-called Penalized composite link models. Composite Link Models are a general framework for modelling generalized linear models, we extend this methodology for the estimation of spatial and temporal latent trends when data are observed at an aggregated level. Latent trends are assumed smooth, and smoothness is achieved with penalized splines and tensor products. The proposed methodology will be illustrated with real examples of mortality data in disease mapping applications such a cardiovascular diseases in females in the Community of Madrid (Spain) and Q-fever disease outbreak in the Netherlands.

This is a joint work with Diego Ayma and María Durbán from Universidad Carlos III de Madrid (Spain).

References

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Fractional operators in action: tackling important modeling challenges in cardiac electrophysiology

Luca Gerardo-Giorda *

Abstract

Classical models of electrophysiology often feature extremely accurate descriptions of the underlying cardiac fiber structure but do not typically account for the effects that high tissue heterogeneities have on electrical pulse propagation, as they emerge from experimental data [?]. The use of space-fractional operators was suggested to overcome such limitations [?]. The crucial issue in using differential operators of fractional order for real world applications is that they are naturally defined on the entire space $\mathbf{R}^n$, $n \geq 1$. However, in the majority of practical cases one needs to model quantities that are defined only on a bounded domain $\Omega \subset \mathbf{R}^n$. The main challenge is hence to suitably restrict, adapt, or interpret the definition of a fractional operator so that it preserves its nonlocal character while allowing for a well-posed formulation of the problem on Ω.

In this talk, I will first describe a novel discretisation of the spectral fractional Laplacian on bounded domains. Relying on the integral formulation of the operator via the heat-semigroup formalism, it can handle at once cartesian domains and irregular geometries [?]. I will then show how this framework allows to combine structural anisotropy and tissue heterogeneity via a nonlocal modification of the classical Monodomain model, obtained by a fractional power of its diffusive term: as the power $s \in (0, 1)$ decreases, nonlocal effects are enhanced and increasing levels of heterogeneity are represented [?]. I will present numerical results on realistic geometries capturing the characteristic features of both anisotropy and heterogeneity.

The talk combines results obtained with N. Cusimano and G. Pagnini (BCAM), F. del Teso (NTNU), and A. Gizzi (Campus Biomedico)

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Goal-oriented adaptivity employing forward-in-time pseudo-dual problems

Judit Muñoz-Matute *David Pardo †Victor M. Calo ‡Elisabete Alberdi §

Abstract

In goal-oriented adaptivity, we adapt the finite element mesh in order to reduce the error of the solution in some quantity of interest [2]. In time-dependent problems, this process involves solving a dual problem that runs backwards in time and that is, in general, computationally expensive [1]. In this work, we define a pseudo-dual problem [3] that runs forwards in time for parabolic problems. We also describe a forward-in-time goal-oriented adaptive algorithm that is suitable for some specific situations (e.g., pure diffusion problems). However, it is not possible in general to define a dual problem running forwards in time that provides information about future states. As an alternative, we also propose a mixed algorithm that combines both the backward-in-time classical dual problem with the forward-in-time pseudo-dual problem. We provide numerical evidence to illustrate the efficiency of both algorithms.

Keywords: linear advection-diffusion equation, goal-oriented adaptivity, pseudo-dual problem, error representation, finite element method

References

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A Painless Automatic *hp*-Adaptive Strategy for Elliptic 1D and 2D Problems: Preliminary Results.

Vincent Darrigrand [*] David Pardo [*†‡] Théophile Chaumont-Frelet [§]
Ignacio Gómez-Revuelto [¶] Luis E. Garcia-Castillo [‖]

Abstract

Despite the existence of several *hp*-adaptive algorithms in the literature (e.g. [1]), very few are used in industrial context due to their high implementational complexity, computational cost, or both.

This occurs mainly because of two limitations associated with *hp*-adaptive methods: (1) The data structures needed to support *hp*-refined meshes are often complex, and (2) the design of a robust automatic *hp*-adaptive strategy is challenging.

To overcome limitation (1), we adopt the multi-level approach of D'Angela et al. [2]. This method handles hanging nodes via a multilevel technique with massive use of Dirichlet nodes.

Our main contribution in this work is intended to overcome limitation (2) by introducing a novel automatic *hp*-adaptive strategy. For that, we have developed a simple energy-based coarsening approach that takes advantage of the hierarchical structure of the basis functions. Given any grid, the main idea consists in detecting those unknowns that contribute least to the energy norm, and remove them. Once a sufficient level of *unrefinement* is achieved, a global h, p, or any other type of refinements can be performed.

We tested and analyzed our algorithm on one-dimensional (1D) and two-dimensional (2D) benchmark cases. The obtained adapted meshes are competitive with the optimal ones shown in [1]. In this presentation, we shall illustrate the main advantages and limitations of the proposed *hp*-adapted method.

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Diffusion approximation in population dynamics

Rolando Rebolledo *

Abstract

The frequency of genes in interconnected populations and of species in interconnected communities are affected by similar processes, such as birth, death and immigration. The equilibrium distribution of gene frequencies in structured populations is known since the 1930s, under Wright's metapopulation model known as the island model. The equivalent distribution for the species frequency (i.e. the species proportional abundance distribution (SPAD), at the metacommunity level, however, is unknown. In this contribution, an open system approach provides a stochastic model to analytically account for this distribution (SPAD). It is shown that the same as for genes SPAD follows a beta distribution, which provides a good description of empirical data and applies across a continuum of scales. This stochastic model, based upon a diffusion approximation, provides an alternative to neutral models for the species abundance distribution (SAD), which focus on number of individuals instead of proportions, and demonstrate that the relative frequency of genes in local populations and of species within communities follow the same probability law.

In addition, we will derive necessary and sufficient conditions for the limit diffusion to have a beta distribution as a steady state.

References

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Just-in-Time Point prediction using a computationally-efficient Lebesgue-sampling-based prognostic method: application to battery end-of-discharge prediction

Camilo Reyes * Francisco Jaramillo † Marcos Orchard ‡
Bin Zhang § Chetan Kulkarni ¶

Abstract

Battery energy systems are becoming increasingly popular and estimating accurately and in real time the moment of total discharge is a key element for energy management. We propose a novel prognostic method based on a combination of classic Riemann sampling and Lebesgue sampling applied to a discharge model of a battery.This combination uses the advantages of the two methods of data sampling. The method consists of an early and inaccurate prediction using a Riemann based sampling method combined with a particle-filter based prognostic. Once a fault condition has been detected, subsequent updates of the Just-in-Time Point (JITP) estimation are done using a novel Lebesgue based sampling method. JITP updates are triggered whenever the Kullback-Leibler divergence between the Riemann based long-term prediction and the posterior (filtered) state PDF differ significantly. Results show that this combined approach is several orders of magnitude faster than the classical prognosis scheme, however less accurate. The combination of these two methods provides a robust result with less computational resources, a key factor to consider in real-time applications and embedded systems.

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Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, 17 y 18 Enero, 2019

Automatic variationally stable finite element formulations

Sergio Rojas *

Abstract

The Discontinuous Petrov-Galerkin (DPG) methods, first introduced by Demkowicz and Gopalakrishnan in [?], are a family of Petrov-Galerkin methods where the test space is taken discontinuous. The main advantages of such methods are that, by construction, they enjoy stability at the discrete level, while also a reliable representation for the error is obtained on the fly, making it a suitable method for mesh-adaptivity based numerical implementations.

In this work, we propose a novel Petrov-Galerkin method that utilizes traditional continuous trial spaces and discontinuous piece-wise polynomial test functions. It can be interpreted in several forms, including: (a) a DPG method with continuous trial spaces (see [?], [?]) or (b) a DG method in which we restrict to continuous trial spaces and add residual minimization. This second interpretation permits us to prove the superior stability properties of the method. Moreover, it provides us with a constructive way to define an adequate inner product for the residual minimization (see [?] for an interpretation of DPG as a residual minimization).

The proposed method, when applied to a first order FE formulation, exhibits two important features: (a) from the implementation point of view, existing FE codes can be easily upgraded to the proposed formulation since no jump terms (as in DG methods) nor trace variables are employed (as in other existing DPG methods like Demkowicz et. al.), (b) the resulting method presents stability properties alike those exhibited by some existing DG or DPG methods (e.g., Demkowicz/Gopalakrishnan).

In this talk, we will briefly introduce the treatment of formulations in strong form. We will also show through numerical examples the efficiency of the method in extreme scenarios where the method gives stable solutions that are comparable with those obtained with classical stabilized methods.

References

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Maxwell Revisited

NORBERTO SÁINZ *

Abstract

This work explores what other mathematical possibilities were available to Maxwell for formulating his electromagnetic field model, by characterizing the family of mathematical models induced by the analytical equations describing electromagnetic phenomena prevailing at that time. The need for this research stems from the article Inertial Relativity A Functional Analysis Review, published in Proyecciones, which claims and demonstrates the existence of an axiomatic conflict between the special and general theories of relativity on one side, and functional analysis on the other, making the reformulation of the relativistic theories, mandatory. As will be shown herein, such reformulation calls for a revision of Maxwells electromagnetic field model. The conclusion is reached that given the set of equations considered by Maxwell not a unique, but an infinite number of mathematically correct reformulations to Ampères law exists, resulting in an equally abundant number of potential models for the electromagnetic phenomena (including Maxwells). Further experimentation is required in order to determine which is the physically correct model.

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Two discontinuous spacetime methods for the linear transport and wave equations

Paulina Sepúlveda *Jay Gopalakrishnan †David Pardo ‡

Abstract

This talk will address the features and limitations of two Petrov-Galerkin spacetime methods. We will first present a spacetime discontinuous Petrov-Galerkin (DPG) method for the linear wave equation. This method is based on an ultraweak formulation that uses a broken graph space [1]. The second method is based on a Discontinuous-in-time Petrov-Galerkin formulation that employs explicit-in-time finite elements [2]. When solving a large time-dependent problem, adaptive mesh refinement schemes become an important tool to obtain efficient numerical simulations. Maintaining the explicit-in-time method while designing a spacetime adaptive mesh refinement scheme still remains a difficult task partially due to strong stability constrains that arise from time-stepping methods.

For the first method will present the conditions that lead to a well-posed weak formulation, and how the built-in estimator of the DPG method is useful for spacetime adaptive refinements for the wave operator. For the second one we propose a new spacetime adaptive mesh refinement technique for an explicit-in-time finite element method, using explicit-in-time basis functions corresponding to explicit schemes (as shown in [2]). Numerical results for the linear wave and transport equation will be presented.

References

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Simultaneous Blow-up in a Multi-species Keller-Segel system: An energy method

Karina Vilches * Eduardo Espejo ** Carlos Conca ***

Resumen

In this work is studied a parabolic-elliptic system that corresponds to a set of drift–diffusion where two–species and two–chemical interact. The multi-species Keller–Segel system that represents such interaction follows,

$$\begin{cases} \partial_t u_1(x,t) = \triangle u_1(x,t) - \chi_{11}\nabla(u_1(x,t)\nabla v_1(x,t)) - \chi_{12}\nabla(u_1(x,t)\nabla v_2(x,t)), \\ \partial_t u_2(x,t) = \triangle u_2(x,t) - \chi_{21}\nabla(u_2(x,t)\nabla v_1(x,t)) - \chi_{22}\nabla(u_2(x,t)\nabla v_2(x,t)), \\ \delta_1\partial_t v_1(x,t) = \triangle v_1(x,t) + \alpha_{11}u_1(x,t) + \alpha_{12}u_2(x,t), \\ \delta_2\partial_t v_2(x,t) = \triangle v_2(x,t) + \alpha_{21}u_1(x,t) + \alpha_{22}u_2(x,t), \end{cases} \tag{1}$$

where t is the time unit and x is the spatial position belong to $\mathbb{R}^2$. The meanings of the variables are: u_i denotes the cell density of type i, which moves diffusively without the presence of a chemo-attractant, and v_j represents the concentration of the chemo-attractant that produces drift forces, for $i,j = 1,2$.

The meanings of the parameters are: χ_{ij} is a positive number representing the *sensitivity* of the cell population to each chemo-attractant, weighting the non-linearity, and α_{ij} denotes the production rate of each cell population, with $i,j = 1,2$. Furthermore, we assume that $\delta_i \approx 0$ for $i = 1,2$. In other words, the time scale in which the chemical molecular diffusion occurs is significantly faster than the diffusion of the cells [15]. Consequently,

$$v_1 = (\alpha_1 u_1 + \alpha_2 u_2) * K \text{ and } v_2 = (\alpha_3 u_1 + \alpha_4 u_2) * K, \tag{2}$$

where $K(\cdot) = -(1/2\pi) * \log|\cdot|$. The initial data $(u_1(x,0), u_2(x,0)) = (u_{10}, u_{20})$ is assumed satisfying the following conditions

$$u_{10}, u_{20} \in L^1(\mathbb{R}^2, (1+|x|^2)dx)\,,\; u_{10}\log u_{10},\; u_{20}\log u_{20} \in L^1(\mathbb{R}^2). \tag{3}$$

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The no-flux conditions at infinity are imposed in such a way that mass conservation can be proved following the arguments presented in [11, p.p.123], and such conditions also permit to apply integration by parts on unbounded domains [4, pp.177].

The system (1) represents interactions between two populations of chemotactic unicellular organisms who produce two different chemical substances, which corresponds to a generalization in $\mathbb{R}^2$ of the model presented in [6, Sec. 2.1] that was formulated to represent the interactions between macrophages and tumour cells in breast cancer. In the case of two chemotactic unicellular populations a new concept arises, the *simultaneous blow-up*. The method presented in this work is useful for compare the behaviour of two chemotactical species using the *free-energy* functional, the entropy, *a priori* L^p estimates, and Alikakos-Moser iterative method extended to $\mathbb{R}^2$. The main results is to obtain the simultaneous Blow-up for a significant zone of parameters in the initial masses plane.

Notice that new questions are arising this problem in the case of several species (larger than 2): How can be interpreted the *simultaneous blow-up*? Could two species present *simultaneous blow-up*, and the others blow-up in different times? Are there other possibles emergent behaviours in the collaborative interaction of several chemotactical species?

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