Strong Large Deviations results for stochastic processes and dynamical systems

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We discuss sufficient conditions that guarantee the existence of asymptotic expansions for Large Deviation Principles. Our focus is on sequences of non-lattice weakly dependent random variables, including strongly ergodic Markov chains, smooth expanding maps and additive functionals of stochastic processes generated from SDE's satisfying the Hőrmander condition. This is joint work with Pratima Hebbar.

Semi-analytical method for stability analysis of periodic orbits in neuronal models

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Abstract :

Spectral Decomposition of Piecewise Contracting Dynamics on the Interval

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We study the topological attractors of injective piecewise contracting maps on a compact interval with any finite number $N \geq 2$ of continuity pieces. We prove the existence of a "spectral decomposition" of the attractor into a finite number of transitive components that are either periodic orbits or Cantor sets. In the non-generic case, we prove that some orbits accumulate at both sides of the discontinuities points, and that this phenomenon generates the transitive Cantor sets of the attractor.

Decay of correlations for suspension flows and Gibbs measures

Diego Daltro [1] and Paulo Varandas[2]

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A major topic of research in ergodic theory is the study of the velocity of mixing, or decay of correlations. In the case of flows the exponential decay of correlations for flows has been considered mainly with respect to the SRB measure for hyperbolic and Lorenz-like attractors. Indeed, SRB measures satisfy the Federer property, which plays a key role in the cancellations argument used by Dolgopyat [6] for proving decay of correlations for flows. Note that the Federer property may fail for Gibbs measures in hyperbolic dynamics (cf. [4]). We discuss exponential decay of correlations for Gibbs measures and suspension semiflows over $C^{1+\alpha}$ (piecewise) expanding, Markov interval maps.

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Graphs Extensions And Amenability

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Abstract:

Kesten's amenability criterion is a classical theorem from probability theory stating that a group is amenable if and only if the Markov operator associated to the symmetric randon walk on the group has spectral radius 1. In this work, we extend this approach to graphs extensions of a full Markov maps (and of a Markov map with embedded Gibbs-Markov struture). That is, we consider the time evolution of the second coordinate of

$$T : X \times \mathcal{G} \to X \times \mathcal{G}, (x, g) \mapsto (f(x), \kappa_x(g)),$$

where $f : X \to X$ is Markov map (or is Markov map with embedded Gibbs-Markov struture), $\kappa_x : V \to V$ is a bijection and constant on cylinders, and a graph $\mathcal{G}$ is an ordered pair $\mathcal{G} = (\mathbf{V}, \mathbf{E})$ where $\mathbf{V}$ referring to the set of vertices and $\mathbf{E}$ to the edges.

We say that $\mathcal{G}$ is a graph amenable if

$$\inf \left\{ \frac{|\partial K|}{|K|} : K \subset \mathbf{V}, |K| < \infty \right\} = 0.$$

With this approach, we prove under some mild assumptions that $\mathcal{G} = (\mathbf{V}, \mathbf{E})$ is amenable if only if $\rho(\hat{T}) = 1$. Where $\hat{T}$ denote a Trasnfer operator.

Continuity of Lyapunov exponents for linear cocycles with a single holonomy

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We consider a fixed hyperbolic dynamic in the base and study how the Lyapunov exponents vary as functions of the cocycle. The continuity of the Lyapunov exponents has been proved by Backes, Brown and Butler for cocycles that admit stable and unstable uniform holonomies. In this talk, we present a partial result of a conjecture of Marcelo Viana that states that a single uniform holonomy is sufficient to guarantee the continuity. This is a joint work with Karina Marin (UFMG).

Spectral view of Hartman-Grobman's theorem for non-uniform and unbounded hyperbolic flows

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Abstract

In this poster it will present that a non-autonomous linear differential system with nonuniform hyperbolicity on the half line can be written as diagonal system with a perturbation which is small enough. Moreover we show that the diagonal terms are contained in the nonuniform exponential dichotomy spectrum. In addition, an example of this result will be shown. On the other hand, it will be shown how this result allows us to establish a bijection between the solutions of the non-autonomous linear differential system (which has non-uniform asymptotic stability) and its unbounded perturbation. For this purpose we introduce the concepts of *nonuniform almost reducibility* and *nonuniform contractibility* in the first part, which are generalization of this notions originally defined in a uniform context, and for the second part, we introduce the concept of *nonuniform topological equivalence*.

Trabajo realizado en conjunto con:
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On A Variational Principle for Regular Impulsive Semiflows

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In [1], was studied a variation of Bowen's topological entropy which can be applied to the study of discontinuous semiflows on compact metric spaces. Moreover, was proved the entropy using separated set of a regular impulsive semiflow agree the usual entropy of a continuous semiflow, if this semiflows are bijective and semiconjugated. The main results of this paper prove that the entropy using spanning set of the regular impulsive semiflow agree the usual entropy of the continuous semiflow and that the regular impulsive semiflows satisfied a variational principle.

[1] Jaque, N., San Martn, B., Topological entropy for discontinuous semiflows, J. Differential Equations 2018

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Controllability, vanishing mean drift and infinite invariant measures

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In the recent paper by D. Burago, S. Ivanov and A. Novikov, "A survival guide for feeble fish", it has been shown that a fish with limited velocity can reach any point in the (possibly unbounded) ocean provided that the fluid velocity field is incompressible, bounded and has vanishing mean drift. This result extends some known global controllability theorems though being substantially nonconstructive. We give a fish a different recipe of how to survive in a turbulent ocean, and show its relationship to structural stability of dynamical systems by providing a constructive way to change slightly the velocity field to produce conservative (in the sense of not having wandering sets of positive measure) dynamics. In particular, this leads to the extension of Ch. Pugh's closing lemma to incompressible vector fields over unbounded domains. The results are based on an extension of the Poincaré recurrence theorem to some σ-finite measures and on specially constructed Newtonian potentials.

Also, we provide a discrete version of our results and prove that systems with small mean drifts satisfy many properties of ones with a probability invariant measure.

For further details and proofs please see the full text at arxiv:1801.06200.

Genericity of Expansive Lorenz Maps with slow recurrence

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In this work we study the behaviour of the singular values of expansive Lorenz maps of the interval. Particularly, the singularities allow the existence of measures with positive entropy, rapid recurrence to the singular region and infinite Lyapunov exponents.

We have been working to verify the genericity of the applications in which the singular values present slow recurrence for themselves. This may allow to show that generically in this set of functions all invariant probabilities with positive entropy have slow recurrences to the singular region and so, all the good candidates to equilibrium states can be lifted to certain induced Markov maps in order to allow the study of the thermodynamic formalism for these applications.

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Commutative Piecewise Linear Maps of Unit Interval
Makar Plakhotnyk

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Let f be the classical tent map $x \mapsto 1 - |1 - 2x|$. We have described early the continuous surjective solutions $\xi : [0,1] \to [0,1]$ of the functional equation $f \circ \xi = \xi \circ f$, which we have called self semi conjugations of f.

Theorem 1. The graph of every self semi conjugation ξ_n of the tent map f linearly extends points $\left\{ \left(\frac{k}{n}, \frac{1+(-1)^{k+1}}{2} \right), 0 \leq k \leq n \right\}$ for some fixed parameter $n \geq 1$ (see [1] and [2]).

The iterations of any surjective unimodal map $g : [0,1] \to [0,1]$ provide examples of its self-semi conjugations. We will present the next fact.

Theorem 2. [3] Let g be a piecewise linear unimodal map of interval $[0,1]$ into itself, which is topologically conjugated with the tent map f. If there exists a piecewise linear self semi conjugacy of g, which is not an iteration of g, then the conjugacy of g and f is piecewise linear. In particular, all self semi conjugacies of g are piecewise linear in this case.

We can use Theorem 2 for a classification of piecewise linear maps, which are topologically conjugated with the tent map via piecewise linear conjugacy.

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Positive Entropy Through Pointwise Dynamics

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Abstract : In this poster we define some pointwise properties of topological dynamical systems and give sufficient conditions for such a system possesses positive topological entropy. We also give sufficient conditions to obtain positive topological entropy for maps which are approximated by maps with the shadowing property in a uniform way. This is a joint work with Alexander Arbieto.

Entropy for N-distal homeomorphisms

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The distal homeomorphisms were introduced by Hilbert (p. 405 [1]) in order to generalize the isometries on metric spaces. Such homeomorphisms have been widely studied in the literature. For instance, Ellis [1] reduced them to the enveloping semi-groups and the minimal distal systems; Fustenberg [2] proved a structure theorem and Parry [4] proved that they have zero entropy (also derived from Fustenberg's). Generalizations of the distal systems include the point distal flows (by Veech [5] who obtained a structure theorem for them) and the more recent N-distal homeomorphisms by Morales in [3]. We will show some dynamic properties of N-distal homemorphisms and we will focus on how it is possible to extend the Parry's result [4] for N-distal homemorphisms.

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