Cristobal Rojas

minicurso: "Introducción a la dinámica computable"
Abstract: La primera parte del curso será una introducción a la teoría de los algoritmos y a la calculabilidad de problemas matemáticos en estructuras contables. En particular, veremos en detalle algunos ejemplos de problemas no calculables. En la segunda parte nos concentraremos en extender la teoría de la calculabilidad a espacios métricos y revisaremos el problema de construir el mapeo de Riemann para un dominio dado, y el problema de encontrar una medida invariante para una dinámica dada.

Charla: "Computational Complexity in Complex Dynamics"
Abstract: In a surprising result on 2006, Braverman and Yampolsky proved the existence of complex parameters which are very quick to compute at any given precision, but for which the associated Julia set is impossible to draw on a computer screen at arbitrary accuracy, even if provided with unlimited time and memory. Many other related results have been obtained since then, which now offer a more global understanding of the variety of complexities arising from the dynamics of this family. In this talk we will try to present an overview of the field, highlighting some important recent results and open problems.

Carolina Canales

Abstract: We will talk about Levi-flat hypersurfaces in complex surfaces. These objects appear in the intersection of complex geometry and complex dynamics. In the first context, they correspond to the boundary of pseudoconvex spaces. In the second context, they have a holomorphic foliation, called Cauchy-Riemann foliation (CR), whose leaves are immersed Riemann surfaces. We are interested in the relations between the dynamics of the CR foliation, the topology of the Levi-flat hypersurface and the geometry of its complement on the ambient surface. We will explore several examples to illustrate them. Finally, we will discuss the ideas to prove these relations and to obtain some new relations.

Esta charla se tratará sobre hipersuperficies Levi-flat en superficies complejas. Estos objetos aparecen en la intersección de la geometría compleja y la dinámica compleja. En el primer contexto son el borde de espacios pseudoconvexos. En el segundo contexto, tienen una foliación holomorfa, llamada foliación de Cauchy-Riemann (CR), cuyas hojas son superficies de Riemann inmersas. Nos interesan las relaciones entre la dinámica de la foliación CR, la topología de la hipersuperficie Levi-flat y la geometría de su complemento en la superficie ambiente. Exploraremos varios ejemplos para ilustrarlas. Finalmente, discutiremos las ideas para probar estas relaciones y para obtener algunas relaciones nuevas.

Patrick Ingram

Title: The critical height and its depleted variants

Abstract: Rational functions of degree d > 1 in one variable are parametrized by a quasi-projective variety. From the point of view of arithmetic geometry, it is natural to study points on this variety through the machinery of Weil heights, while the dynamical interpretation suggests other measures of complexity, such as the "critical height" introduced by Silverman. This talk will present some recent work relating the critical height to Weil heights on moduli space, and then go on to suggest some further directions involving variants of the critical height.

Hongming Nie

Title: Iteration at the Boundary of Newton's Method

Abstract: We study the iterate maps restricted on the moduli spaces of Newton's method. We prove these restrictions extend continuously to the GIT compactifications of moduli spaces of Newton's method. Moreover, we give complete descriptions of rescaling limits for holomorphic families of Newton's method.

Felipe Riquelme

Title: Amount of failure of upper-semicontinuity of entropy for the geodesic flow on hyperbolic surfaces

Abstract: The entropy of a dynamical system is an important conjugacy invariant that, roughly speaking, allows to understand chaos. From the measure-theoretical point of view, it is known that entropy only can increase when considering weak*-limits of probability invariant measures on smooth systems

defined on compact Riemannian manifolds. However, to the best of our knowledge, once we admit the manifold to be non-compact there are no such general statement.
In this talk we will study the geodesic flow on the unit tangent bundle of finitely generated hyperbolic surfaces. We will estimate the amount of failure of upper-semicontinuity of the measure-theoretical entropy by taking in consideration the escape of mass phenomena and the geometry of the non-compact part of such surfaces. We will conclude that measures with large enough entropy cannot lose the whole mass at the limit. We will also give enough arguments in order to generalize these results to the context of geometrically finite Riemannian manifolds with pinched negative sectional curvature.

Peter Veerman

4 Mediatrices and Minimal Separating Sets

For distinct points p and q in a two-dimensional connected Riemannian manifold M, we define their *mediatrix* L_{pq} as the set of points equidistant to p and q. It is known that mediatrices have a cell decomposition consisting of a finite number of branch points connected by Lipschitz curve. We show additional geometric regularity properties of mediatrices: at each point they have the radial linearizability property, which means that they are tangent to a finite collection of lines meeting in the origin. Simply put: they are Lipschitz images of (multi) graphs. In the case of mediatrices on the sphere, where mediatrices are simple closed Lipschitz curves, we show these curves have at most countable singularities, and the total angular deficiency has a finite upper bound related to the total curvature of the metric on the sphere.

On the other hand mediatrices have the *minimal separating* property: they separate the manifold M into two parts and that any proper subset of them does not. This fact allows for their topological classification. In principle we can determine which (multi)graphs can minimally separate a surface of genus g. This classification is in some sense a generalization of the Jordan Brouwer Theorem. We will briefly discuss the classification of minimal separating sets in the orientable surfaces of genus 0, 1, 2, and 3.

Mediatrices found an application in a long-standing territorial conflict between Peru and Chile. In 2014, the International Court of Justice in The Hague weighed in on the issue and used the concept of mediatrix in its decision. We will show what they did.

Decidability and arithmetical dynamic of quadratic polynomials

Marianela Castillo
Universidad de Concepción

October 5, 2017

X. Vidaux and C. R. Videla in [2] prove that certain towers $\mathbb{Z}^{(\nu,x_0)}$ of totally real algebraic integers satisfy a topological property which implies that they have theory an undecidable theory.

In this work we show that, if the constant term of an iterated quadratic polynomial associated to the tower remains square-free, then the ring $\mathbb{Z}^{(\nu,x_0)}$ is the ring of integers of its fractions field. We will explain how this would answer an old question of Julia Robinson.

References

[1] M. Castillo, *Characterizing intervals containing complete sets of conjugates in a family of totally real towers of nested square roots*, preprint.

[2] X. Vidaux and C. R. Videla, *Definability of the natural numbers in totally real towers of nested square roots*, Proc. Amer. Math. Soc. **143**, 4463–4477 (2015).

Encuentro Anual de Geometría Compleja
y
Dinámica Holomorfa-Aritmética

27-29 de Noviembre, 2017, PUCV, Valparaíso, Chile

Cuerpos de definición de funciones racionales

R. A. Hidalgo *†

Abstract

Due to Milnor, the moduli space M_d, of rational maps of degree $d \geq 2$, is known to have the structure of a complex orbifold of dimension $2(d-1)$. The **branch locus** $\mathcal{B}_d \subset \mathrm{M}_d$ is given by the classes of rational maps with non-trivial group of holomorphic automorphisms. Miasnikov, Stout and Williams have recently observed that every finite group of $\mathrm{PSL}_2(\mathbb{C})$ can be seen as the full group of holomorphic automorphisms of a suitable rational map and a description of these maps is provided in terms of the corresponding finite group. Milnor also noted, by using the symmetric forms in the multipliers of the fixed points, that M_2 can be identified with $\mathbb{C}^2$. In this model, Fujimora noted that $\mathcal{B}_2$ corresponds to the cubic curve

$$2x^3 + x^2y - x^2 - 4y^2 - 8xy + 12x + 12y - 36 = 0.$$

If $d \geq 3$, $\mathcal{B}_d$ corresponds exactly to the sublocus of M_d where it fails to be a topological manifold. We observe that the branch locus $\mathcal{B}_d$ is always connected

The conjugation action on rational maps by the reflection $J(z) = \overline{z}$ provides of a **real structure** on M_d. The locus $\mathrm{M}_d^{\mathbb{R}}$ of real points of such a structure consists to those rational maps admitting antiholomorphic automorphisms. If a rational maps admits a reflection (antiholomorphic involution having fixed points) as an automorphism, then it is called **real**. A rational maps admitting antiholomorphic automorphisms, but none of them being a reflection, is called **pseudo-real**. The locus $\mathrm{M}_d(\mathbb{R}) \subset \mathrm{M}_d$ consisting of real rational maps is a connected real orbifold of real dimension $2(d-1)$. If we denote by $\mathcal{P}_d \subset \mathrm{M}_d$ the locus consisting of the pseudo-real rational maps, then $\mathrm{M}_d^{\mathbb{R}}$ is the disjoint union of $\mathrm{M}_d(\mathbb{R})$ and $\mathcal{P}_d$. Silverman noted that $\mathcal{P}_d = \emptyset$ if d is even and that, for $d \geq 3$ odd, $\mathcal{P}_d \neq \emptyset$. We show that (for $d \geq 3$ ood) the locus $\mathcal{P}_d$ is always disconnected and that $\mathrm{M}^{\mathbb{R}}$ is always connected. We also have provide a description of the rational maps admitting an antiholomorphic automorphism.

Given a rational map R, there is an associated field $\mathcal{M}_R$, called its **field of moduli** (this is an invariant under the action of the group $\mathrm{Gal}(\mathbb{C})$). By results due to Koizumi, $\mathcal{M}_R$ is the intersection of all the fields of definitions of R. Silverman observed that, for either d even or R equivalent to a polynomial, this field is a field of definition of R. In the case that R cannot be defined over its field of moduli (so $d \geq 3$ must be odd), we have seen that it can be defined over a suitable quadratic extension of it.

The field $\mathcal{M}_R$ is a subfield of $\mathbb{R}$ if and only if R is either real or pseudo-real. Also, R can be defined over the reals if and only if it is real. In particular, pseudo-real rational maps are examples of rational maps not definable over their field of moduli. Explicit examples of pesudo-real rational maps, with trivial group of holomorphic automorphisms were provided by Silverman.

*Partially supported by FONDECYT 1150003 and Anillo ACT 1415 PIA-CONICYT
†e-mail: ruben.hidalgo@ufrontera.cl

We observe that, for a pseudo-real rational map, its group of holomorphic automorphisms is either trivial or a cyclic group. For each $n \geq 2$, we present explicit examples of pseudo-real rational maps with a cyclic group of order n as group of holomorphic automorphisms.

In the case of real maps, it is more difficult to check if can or not be defined over its field of moduli. We present explicit examples of real rational maps which cannot be defined over the filed of moduli.

VARIEDADES ABELIANAS CON ACCIÓN DE GRUPO

SEBASTIÁN REYES CAROCCA

Abstract. Una variedad abeliana es un toro complejo que es, además, una variedad algebraica. En esta charla discutiremos principalmente de acciones de grupos finitos sobre variedades abelianas, y presentaremos algunos resultados recientes obtenidos por el expositor en conjunto con Rubí E. Rodríguez.

Departamento de Matemática y Estadística, Universidad de La Frontera, Temuco, Chile
E-mail address: sebastian.reyes@ufrontera.cl

Parcialmente financiado por Proyecto Fondecyt Postdoctoral 3160002 y Proyecto Anillo ACT1415 PIA-Conicyt.

Rational maps not definable over their field of moduli

Saúl Quispe*
Departamento de Matemática y Estadística
Universidad de La Frontera
Temuco, Chile

Abstract

The known (explicit) examples of rational maps not definable over their field of moduli have automorphisms group trivial and are not real but their field of moduli is a subfield of the reals [2]. In this talk we provide examples of non-real rational maps with automorphisms group a cyclic group and real rational maps with automorphisms group trivial which cannot be defined over their field of moduli [1].

Joint work with:
Ruben Hidalgo[1], Departamento de Matemática y Estadística, Universidad de La Frontera, Temuco, Chile.

References

[1] R. A. Hidalgo and S. Quispe. On real and pseudo-real rational maps, preprint 2017.

[2] J. H. Silverman. The field of definition for dynamical systems on $\mathbb{P}^1$. *Compositio Mathematica* **98** No. 3 (1995), 269–304.

*Partially supported by Proyecto FONDECYT Iniciación en Investigación N. 11170129 and Proyecto Anillo ACT 1415 PIA CONICYT, e-mail: saul.quispe@ufrontera.cl

[1]Partially supported by Proyecto FONDECYT Regular N. 1150003 and Proyecto Anillo ACT 1415 PIA CONICYT, e-mail: ruben.hidalgo@ufrontera.cl