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Multiscale Hybrid-Mixed Method: An Overview and Recent Developments

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ABSTRACT

We present an overview and some recent results of a new family of finite element methods for multiscale problems, named Multiscale Hybrid-Mixed (MHM) methods. MHM methods are a consequence of a hybridization procedure, which characterizes the unknowns as a direct sum of a "coarse" solution and the solutions to problems with Neumann boundary conditions driven by the multipliers. As a result, the MHM method becomes a strategy that naturally incorporates multiple scales while providing solutions with high-order precision for the primal and dual variables. The completely independent local problems are embedded in the upscaling procedure, and computational approximations may be naturally obtained in a parallel computing environment. Also interesting is that the dual variable preserves the local conservation property using a simple post-processing of the primal variable. Well-posedness and best approximation results for the one- and two-level versions of the MHM method show that the method achieves optimal convergence with respect to the mesh parameter and is robust in terms of (small) physical parameters. Also, a face-based *a posteriori* estimator is shown to be locally efficient and reliable with respect to the natural norms. The general framework is illustrated for the second order elliptic Darcy and the linear elasticity equations, and then further extended to other operators. Numerical results verify the optimal convergence properties as well as the capacity of the MHM method to accurately incorporate heterogeneity and high-contrast coefficients, and the great performance of the new a posteriori error estimator in driving mesh adaptativity.