A monotone+skew splitting model for composite monotone inclusions in duality *

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Abstract

Several problems in engineering, computer science, image processing, among other fields, can be modeled via the following optimization problem defined in a real Hilbert space $\mathcal{H}$:

$$\underset{x\in\mathcal{H}}{\text{minimize}}\ \ f(x)+g(Lx), \tag{1}$$

where $f\colon\mathcal{H}\to\ ]-\infty,+\infty]$ and $g\colon\mathcal{G}\to\ ]-\infty,+\infty]$ are lower semicontinuous convex functions, $\mathcal{G}$ is a real Hilbert space, and $L\colon\mathcal{H}\to\mathcal{G}$ is linear and bounded. Sometimes it is easier to deal with its dual:

$$\underset{v\in\mathcal{G}}{\text{minimize}}\ \ f^*(v)+g^*(-L^*v), \tag{2}$$

where $f^*\colon u\mapsto\sup_{x\in\mathcal{H}}\langle x\mid v\rangle-f(x)$ is the Fenchel conjugate of f and L^* is the adjoint of L. In this talk we propose an algorithm for solving, simultaneously, both problems in a more general setting. Indeed, by using a generalization of the gradient for nonsmooth functions called *subdifferential*, denoted by ∂f (or ∂g) it is possible, under mild assumptions, to formulate both problems via the primal inclusion

$$\text{find}\ \ x\in\mathcal{H}\quad\text{such that}\quad 0\in\partial f(x)+L^*\partial g(Lx) \tag{3}$$

together with the dual inclusion

$$\text{find}\ \ v\in\mathcal{G}\quad\text{such that}\quad 0\in -L\partial f^*(-L^*v)+\partial g^*(v). \tag{4}$$

We will show that these inclusions are equivalent to

$$\text{find}\ \ (x,v)\in\mathcal{H}\oplus\mathcal{G}\quad\text{such that}\quad (0,0)\in\boldsymbol{M}(x,v)+\boldsymbol{S}(x,v), \tag{5}$$

for an appropriate choice of a maximally monotone operator $\boldsymbol{M}\colon\mathcal{H}\oplus\mathcal{G}\to 2^{\mathcal{H}\oplus\mathcal{G}}$ and a linear skew-adjoint operator $\boldsymbol{S}\colon\mathcal{H}\oplus\mathcal{G}\to\mathcal{H}\oplus\mathcal{G}$.

We propose a fully descomposed method for solving simultaneously (3) and (4), which is derived from a splitting method for solving (5). The convergence of the proposed method is established and some applications are examined. The results can be found in [1].

References

[1] L. M. Briceño-Arias and P. L. Combettes, *A monotone+skew splitting model for composite monotone inclusions in duality*, SIAM J. Optim., vol. 21, pp. 1230–1250, 2011.

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